Objective Problems (Level 2)

1. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be
   (a) $\frac{K^2}{K^2+R^2}$ (b) $\frac{R^2}{K^2+R^2}$
   (c) $\frac{K^2+R^2}{R^2}$ (d) $\frac{K^2}{R^2}$

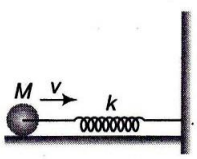
2. A particle of mass m is projected with velocity u at an angle of θ with the horizontal. The initial angular momentum of the particle about the highest point of its trajectory is equal to
   (a) $\frac{mu^3 \sin^2\theta\cos\theta}{3g}$ (b) $\frac{3mu^3 \sin^2\theta\cos\theta}{2g}$
   (c) $\frac{mu^3 \sin^2\theta\cos\theta}{2g}$ (d) None of these

3. A uniform rod of mass 2 kg and length 1 m lies on a smooth horizontal plane. A particle of mass 1 kg moving at a speed of 2 m/s perpendicular to the length of the rod strikes it at a distance $\frac{1}{4}$ m from the centre and stops. What is the angular velocity of the rod about its centre just after the collision?
   (a) 3 rad/s (b) 4 rad/s
   (c) 1 rad/s (d) 2 rad/s

4. A cord is wound around the circumference of wheel of radius r. The axis of the wheel is horizontal and MI is I. A weight mg is attached to the end of the cord and falls from rest. After falling through a distance h, the angular velocity of the wheel will be
   (a) $\sqrt{\frac{2gh}{I+mr^2}}$ (b) $\left(\frac{2mgh}{I+mr^2}\right)^{1/2}$
   (c) $\left(\frac{2mgh}{I+2mr^2}\right)^{1/2}$ (d) $\sqrt{2gh}$

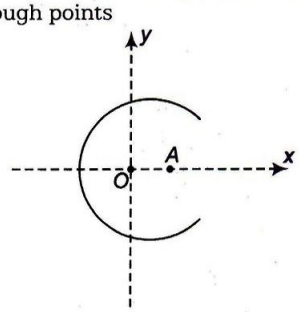
5. The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height h, from rest without sliding is
   (a) $\sqrt{gh}$ (b) $\sqrt{\left(\frac{g}{5}\right)gh}$
   (c) $\sqrt{\left(\frac{4}{3}\right)gh}$ (d) $\sqrt{\left(\frac{10}{7}\right)gh}$

6. A solid sphere rolls without slipping and presses a spring of spring constant k as shown in figure. Then, the compression in the spring will be



   (a) $v\sqrt{\frac{2M}{3k}}$ (b) $v\sqrt{\frac{2M}{5k}}$
   (c) $v\sqrt{\frac{5k}{7M}}$ (d) $v\sqrt{\frac{7M}{5k}}$

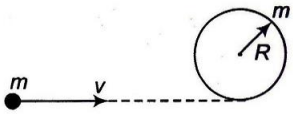
7. A portion of a ring of radius R has been removed as shown in figure. Mass of the remaining portion is m. Centre of the ring is at origin O. Let I_A and I_O be the moment of inertias passing through points



   A and O and perpendicular to the plane of the ring. Then,
   (a) $I_O = mR^2$ (b) $I_O = I_A$
   (c) $I_O > I_A$ (d) $I_A > I_O$

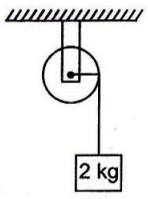
8. Consider three solid spheres, sphere (i) has radius r and mass m, sphere (ii) has radius r and mass $3m$, sphere (iii) has radius $3r$ and mass m. All can be placed at the same point on the same inclined plane, where they will roll without slipping to the bottom. If allowed to roll down the incline, then at the bottom of the incline
   (a) sphere (i) will have the largest speed
   (b) sphere (ii) will have the largest speed
   (c) sphere (ii) will have the largest kinetic energy
   (d) all the spheres will have equal speeds

9. A circular disc of mass m and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m and moving with a velocity v, strikes the disc and gets embedded in it.



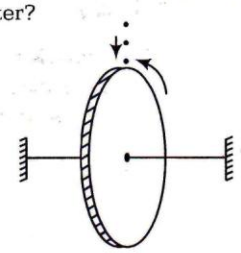
   The angular velocity with which the system rotates after the bullet strikes the hoop is
   (a) $\frac{v}{2R}$ (b) $\frac{v}{3R}$
   (c) $\frac{2v}{3R}$ (d) $\frac{3v}{4R}$

10. A small pulley of radius 20 cm and moment of inertia 0.32 kg-m^2 is used to hang a 2 kg mass with the help of massless string. If the block is released, for no slipping condition acceleration of the block will be



   (a) 2 m/s^2 (b) 4 m/s^2
   (c) 1 m/s^2 (d) 3 m/s^2

11. A disc of mass m_0 rotates freely about a fixed horizontal axis through its centre. A thin cotton pad is fixed to its rim, which can absorb water?



The mass of water dripping onto the pad is μ per second. After what time will the angular velocity of the disc get reduced to half of its initial value?

(a) $\frac{2m_0}{\mu}$ (b) $\frac{3m_0}{\mu}$ (c) $\frac{m_0}{\mu}$ (d) $\frac{m_0}{2\mu}$

12. Four identical rods each of mass m and length l are joined to form a rigid square frame. The frame lies in x-y plane, with its centre at the origin and the sides parallel to the x and y-axes. Its moment of inertia about (more than one may be correct)

(a) the x-axis is $\frac{2}{3}ml^2$

(b) the z-axis is $\frac{4}{3}ml^2$

(c) an axis parallel to the z-axis and passing through a corner is $\frac{10}{3}ml^2$

(d) one side is $\frac{5}{3}ml^2$

13. A solid sphere and a solid cylinder of same mass are rolled down on two inclined planes of heights h_1 and h_2 respectively. If at the bottom of the plane the two objects have same linear velocities, then the ratio of $h_1 : h_2$ is

(a) 2 : 3 (b) 7 : 5 (c) 14 : 15 (d) 15 : 14

14. If I_1 is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass and I_2 is the moment of inertia of the ring about an axis perpendicular to plane of ring and passing through its centre formed by bending the rod, then

(a) $\frac{I_1}{I_2} = \frac{3}{\pi^2}$ (b) $\frac{I_1}{I_2} = \frac{2}{\pi^2}$

(c) $\frac{I_1}{I_2} = \frac{\pi^2}{2}$ (d) $\frac{I_1}{I_2} = \frac{\pi^2}{3}$

15. A horizontal disc rotates freely about a vertical axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, the two rotate with a common angular velocity

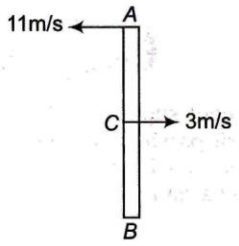
(a) some friction exists between the disc and the ring

(b) the angular momentum of the disc plus ring is conserved

(c) the final common angular velocity is $\frac{2}{3}$rd of the initial angular velocity of the disc

(d) $\frac{2}{3}$ rd of the initial kinetic energy is converted into heat

16. A uniform rod AB of length 7m is undergoing combined rotational and translational motion such that at some instant of time, velocities of its end points A and centre C are both perpendicular to the rod and opposite in direction, having magnitudes 11 m/s and 3 m/s respectively as shown in the figure.



Velocity of centre C and angular velocity of the rod remain constant

(a) acceleration of point A is 56 m/s^2

(b) acceleration of point B is 56 m/s^2

(c) at the instant shown in the figure acceleration of point B is more than that of point A

(d) angular velocity of the rod is 4 rad/s

17. A rod of length L whose lower end is fixed along the horizontal plane starts to topple from the vertical position. The velocity of the upper end of the rod when it hits the ground is

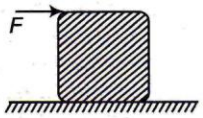
(a) $\sqrt{3gL}$ (b) $\sqrt{2gL}$ (c) $\sqrt{gL}$ (d) $\sqrt{5gL}$

18. A disc of mass m and radius R is rolling on horizontal ground with linear velocity v. What is the angular momentum of the disc about an axis passing through bottommost point and perpendicular to the plane of motion?

(a) $\frac{3}{2}mvR$ (b) mvR

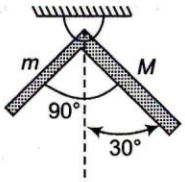
(c) $\frac{1}{2}mvR$ (d) $\frac{4}{3}mvR$

19. A force F is applied on the top of a cube as shown in figure. The coefficient of friction between the cube and the ground is μ. If F is gradually increased, the cube will topple before sliding if



(a) $\mu > 1$ (b) $\mu < \frac{1}{2}$ (c) $\mu > \frac{1}{2}$ (d) $\mu < 1$

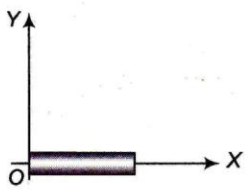
20. Two uniform rods of equal length but different masses are rigidly joined to form an L-shaped body, which is then pivoted as shown in figure. If in equilibrium the body is in the shown configuration, ratio M/m will be



(a) 2 (b) 3

(c) $\sqrt{2}$ (d) $\sqrt{3}$

21. The figure shows a uniform rod lying along the x-axis. The locus of all the points lying on the x-y plane, about which the moment of inertia of the rod is same as that about O is



(a) an ellipse (b) a circle

(c) a parabola (d) a straight line

22. Two discs have same mass and thickness. Their materials are of densities d_1 and d_2. The ratio of their moments of inertia about an axis passing through the centre and perpendicular to the plane is

(a) $d_1 : d_2$ (b) $d_2 : d_1$

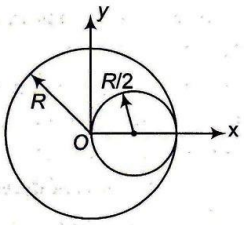
(c) $\left(\frac{d_1}{d_2}\right)^2$ (d) $\left(\frac{d_2}{d_1}\right)^2$

23. A solid sphere of mass 2 kg rolls up a 30° incline with an initial speed of 10 m/s. The maximum height reached by the sphere is ($g = 10$ m/s^2)

(a) 3.5 m (b) 7.0 m

(c) 10.5 m (d) 14.0 m

24. A hole of radius $R/2$ is cut from a thin circular plate of radius R and mass M. The moment of inertia of the plate about an axis through O perpendicular to the x-y plane (*i.e.*, about the z-axis) is



(a) $\frac{5}{7} MR^2$ (b) $\frac{7}{12} MR^2$

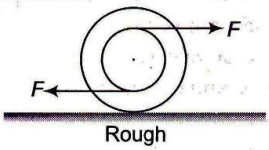
(c) $\frac{13}{32} MR^2$ (d) $\frac{13}{24} MR^2$

25. If a disc of mass m and radius r is reshaped into a ring or radius $2r$, the mass remaining the same, the radius of gyration about centroidal axis perpendicular to plane goes up by a factor of

(a) $\sqrt{2}$ (b) 2

(c) $2\sqrt{2}$ (d) 4

26. A spool is pulled horizontally by two equal and opposite forces as shown in the figure. Which of the following statements is correct?



(a) The centre of mass moves towards left

(b) The centre of mass moves towards right

(c) The centre of mass remains stationary

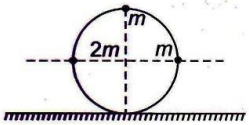
(d) The net torque about the centre of mass of the spool is zero.

27. Two men each of mass m stand on the rim of a horizontal circular disc, diametrically opposite to each other. The disc has a mass M and is free to rotate about a vertical axis passing through its centre of mass. Each mass start simultaneously along the rim clockwise and reaches their original starting points on the disc. The angle turned through by the disc with respect to the ground (in radian) is

(a) $\frac{8m\pi}{4m + M}$ (b) $\frac{2m\pi}{4m + M}$

(c) $\frac{m\pi}{M + m}$ (d) $\frac{4m\pi}{2M + m}$

28. A ring of mass m and radius R has three particles attached to the ring as shown in the figure. The centre of the ring has a speed v_0. The kinetic energy of the system in case of no slipping is



(a) $6m v_0^2$ (b) $12m v_0^2$

(c) $4mv_0^2$ (d) $8mv_0^2$